Variations of a geometric pattern from the Blue Mosque: A practical ruler and compass construction method

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Pattern on the floor of the entrance to the Blue Mosque in Istanbul

Introduction

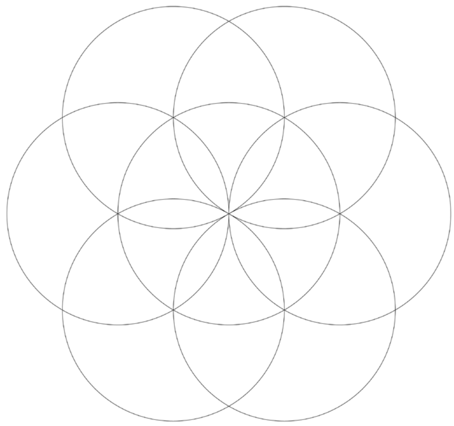
Inspired by Professor Miroslaw Majewski's exploration[1] of the pattern above, this paper presents a practical ruler and compass construction method which allows for the creation of a set of tiles which can be configured in numerous ways, giving rise to a whole range of patterns. Working through this method gives an insight into how a subgrid of regular polygons can underpin the construction of Islamic patterns and provide us with a key for the development of further patterns.

1. Constructing the subgrid

First we will create a subgrid of regular hexagons, squares and triangles. This is a classic Archimedean or semi-regular tiling and provides a repeating structure which can help us analyse the underlying construction of many patterns.

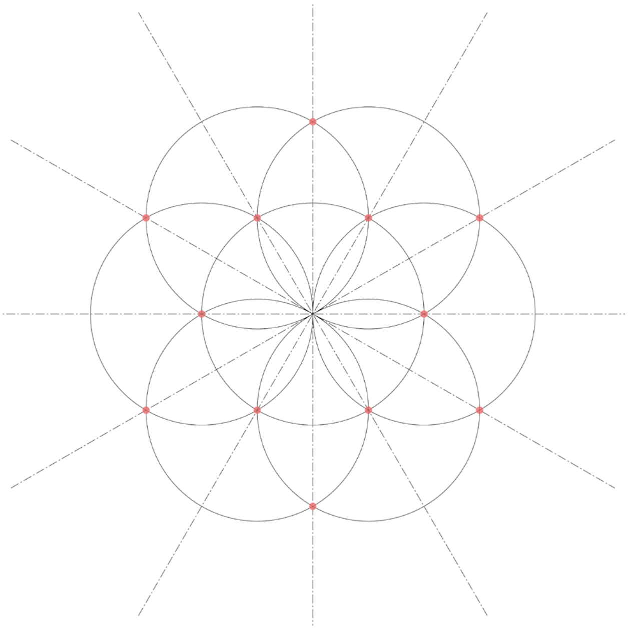
1.

Start with the creation pattern where six overlapping circles of equal radius are centred on the circumference of the central circle.



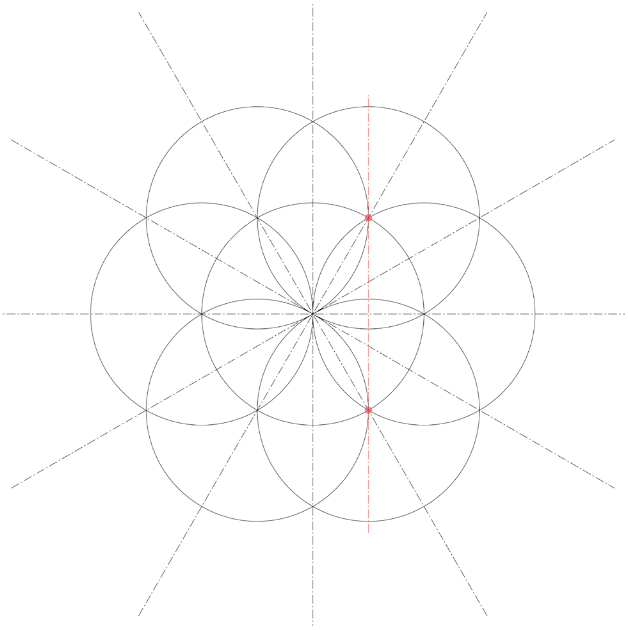
2.

Use the marked intersections to create 12 equal divisions.



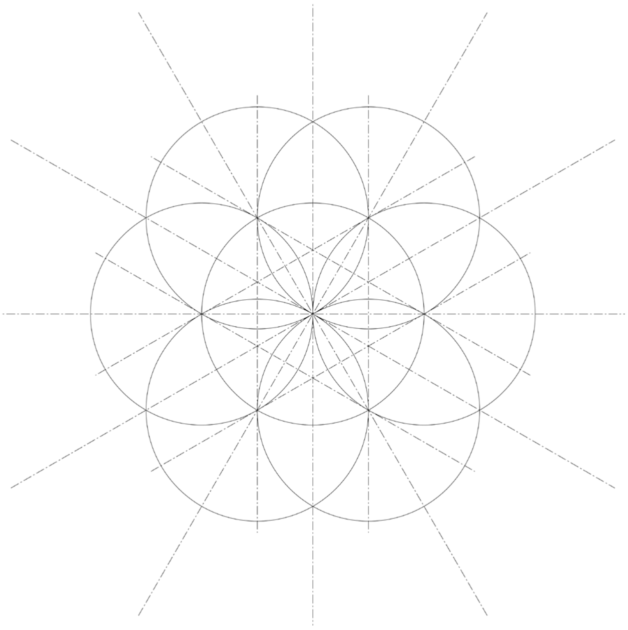
3.

Join the tips of the petals as shown and extend the line to the outer circumference.



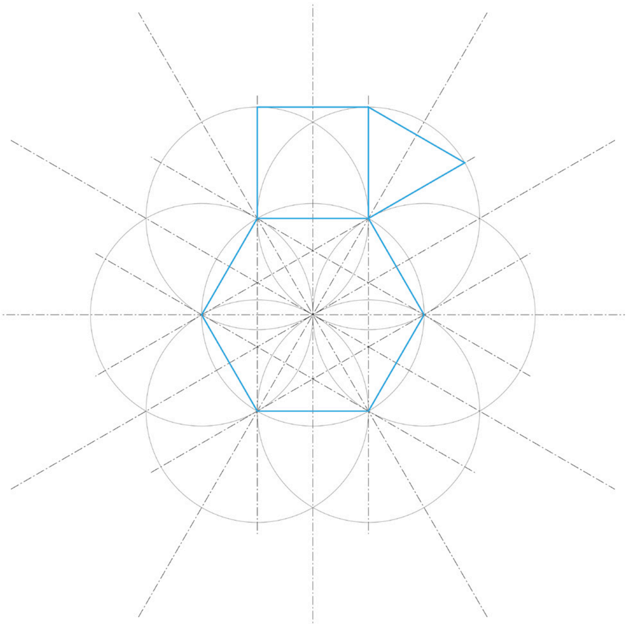
4.

Do the same 5 more times around the circle.



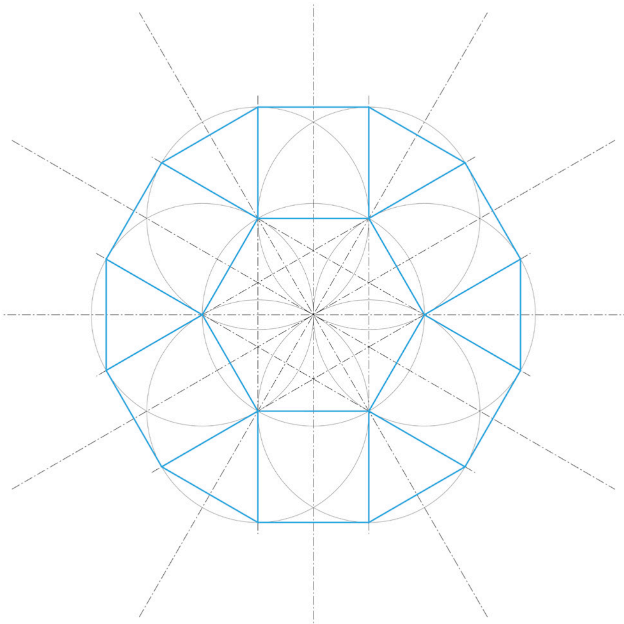
5.

These are the 3 regular polygons that make up the structure of the subgrid; a central hexagon surrounded by triangles and squares.



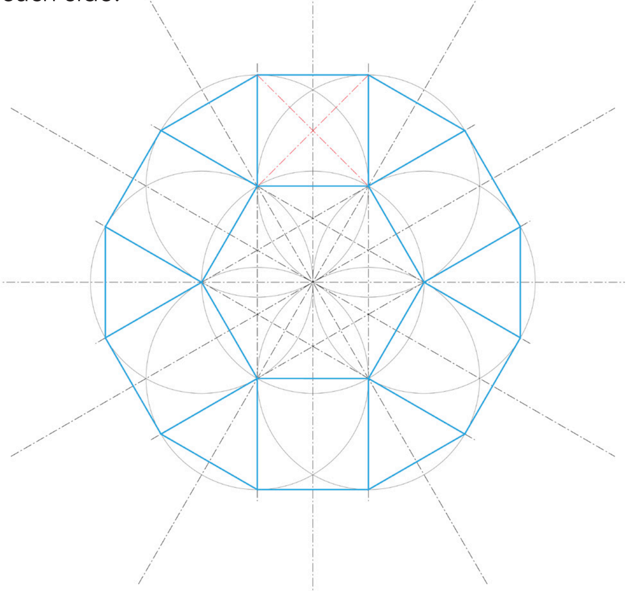
6.

The completed subgrid.



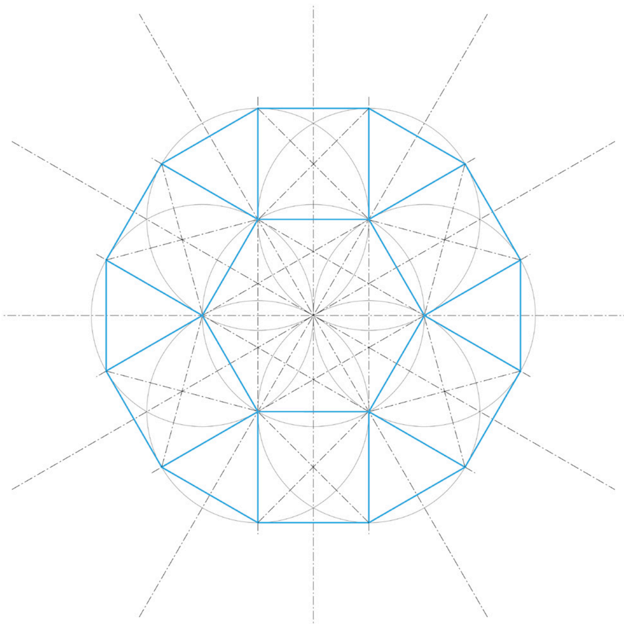
7.

A few more construction lines are required to divide the polygons equally and find the midpoints of each side.



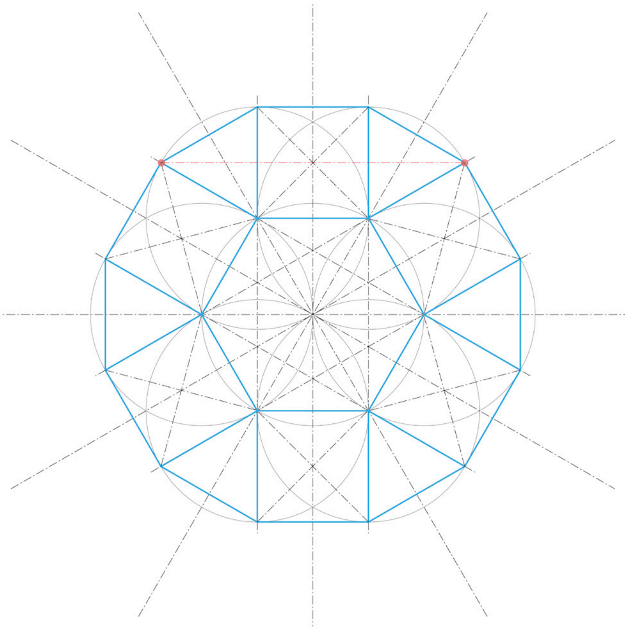
8.

Join the opposite corners of each square.



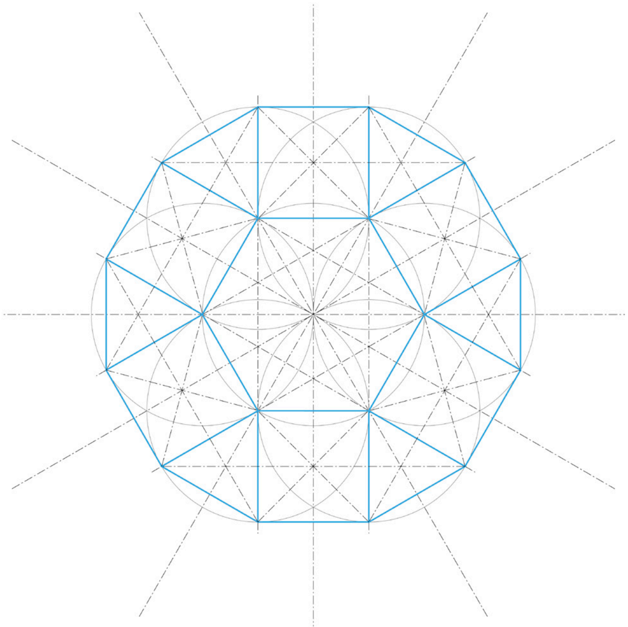
9.

Join the tips of opposing triangles to find the mid points of the remaining sides of the squares. Apply around the figure.



10.

We now have a completed subgrid with the centre point of each side of each polygon marked.

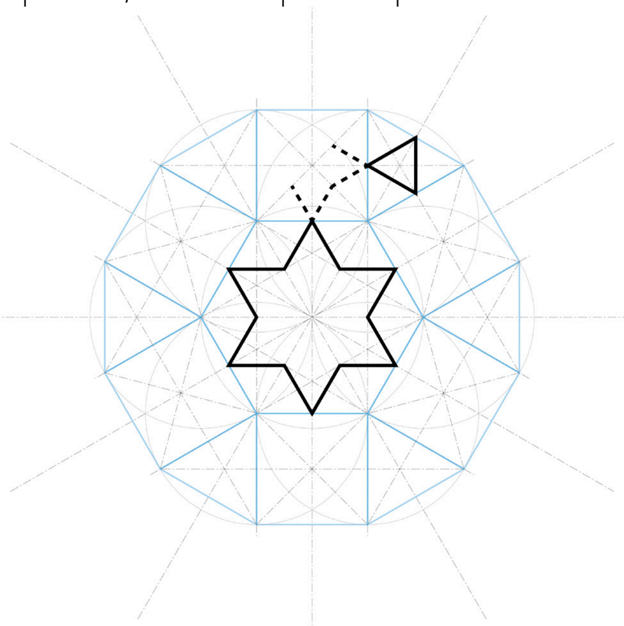


2. Drawing a variation where lines cross at the polygon midpoints

The first pattern we will draw is generated by lines crossing the midpoints of each polygon edge at 60° and extending until they meet another line. This method is referred to as Hankin's polygons in contact method [2].

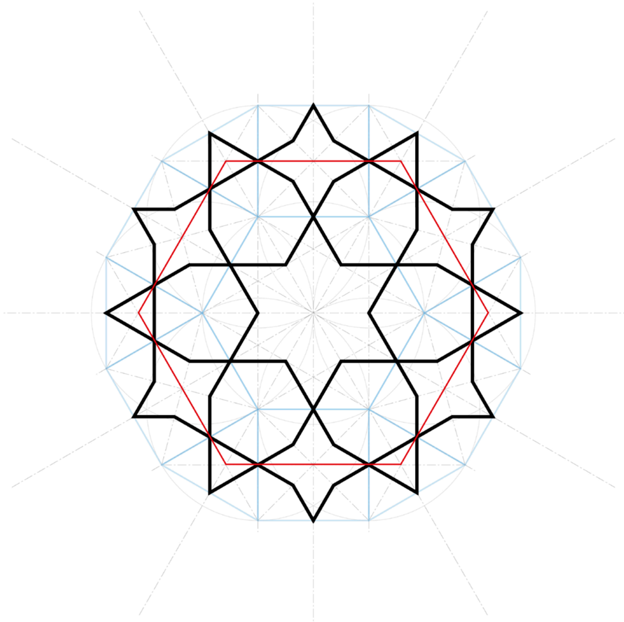
1.

By drawing the six-pointed star and the triangle, and extending the lines to the diagonals of the square tile, we can complete the pattern.



2.

The finished pattern. The red hexagon shows the hexagonal repeat unit.

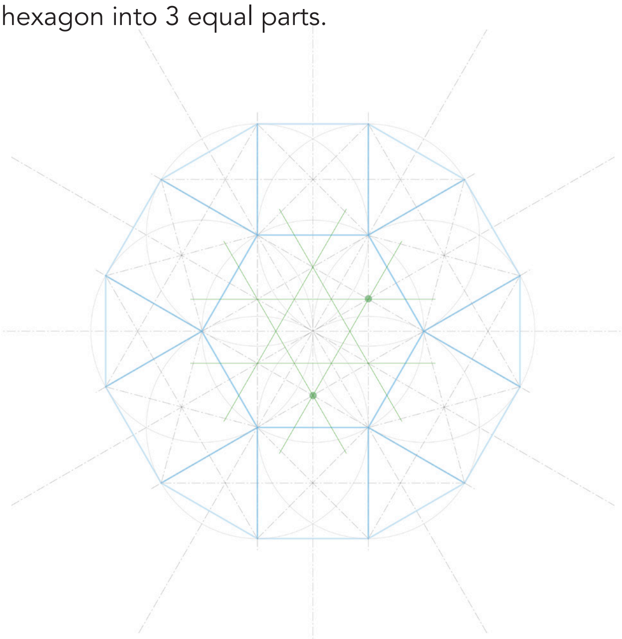


3. Developing the construction to divide the polygon edges into three

We will now divide each polygon edge into three and develop another related pattern where the lines cross at these new points.

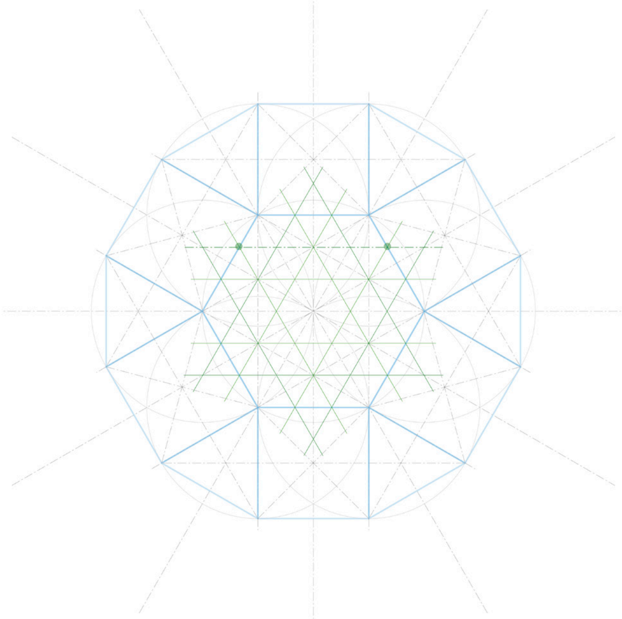
1.

Join the intersections marked here and extend the lines as shown. This divides each edge of the hexagon into 3 equal parts.



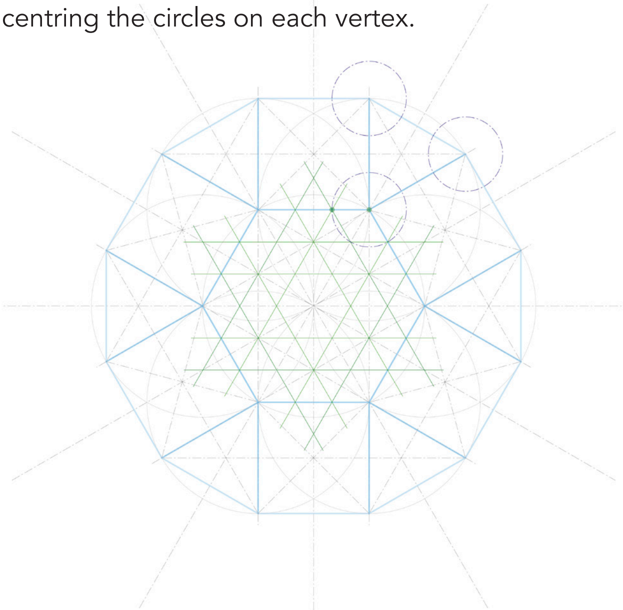
2.

Again, join the intersections shown and apply around the figure.



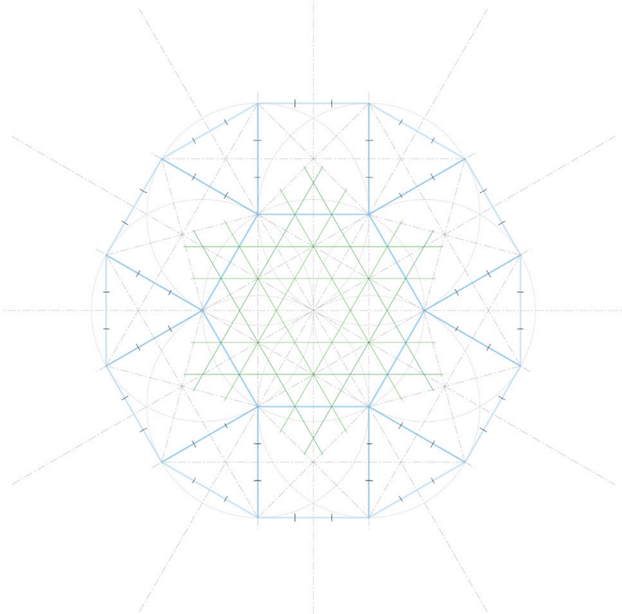
3.

We can use the radial measurement shown below to find the 1/3 point of each remaining side, by centring the circles on each vertex.



4.

There is no need to draw the whole circle, just a mark on each polygon edge will suffice.

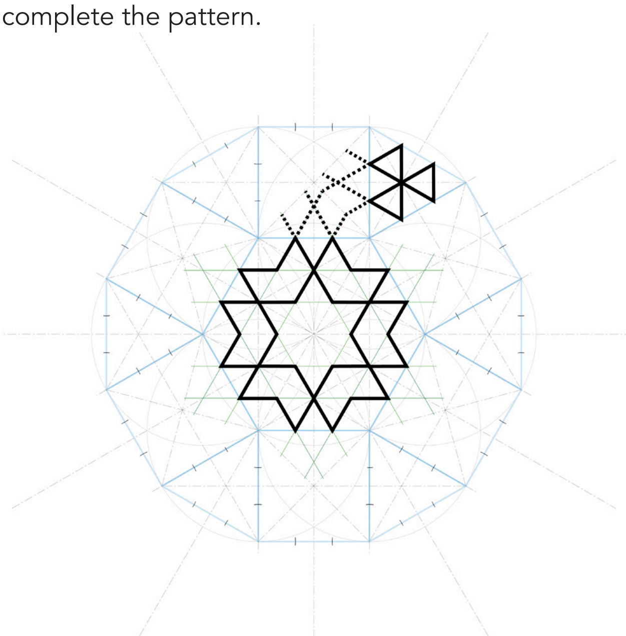


4. Drawing the two-point variation

We now have the necessary information to construct the final design. This variation involves lines crossing the subgrid polygons at two points equidistant along each edge.

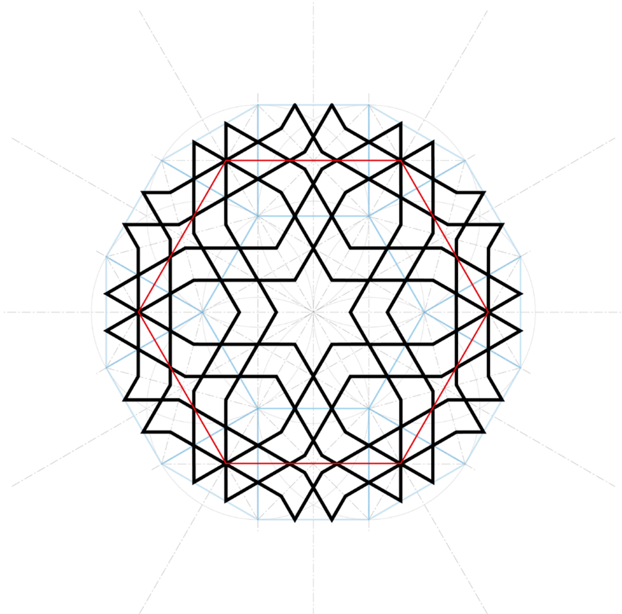
1.

The design in the central hexagon and the triangles can easily be filled in. Extending these lines up to the diagonals inside the square tile allows us to complete the pattern.



2.

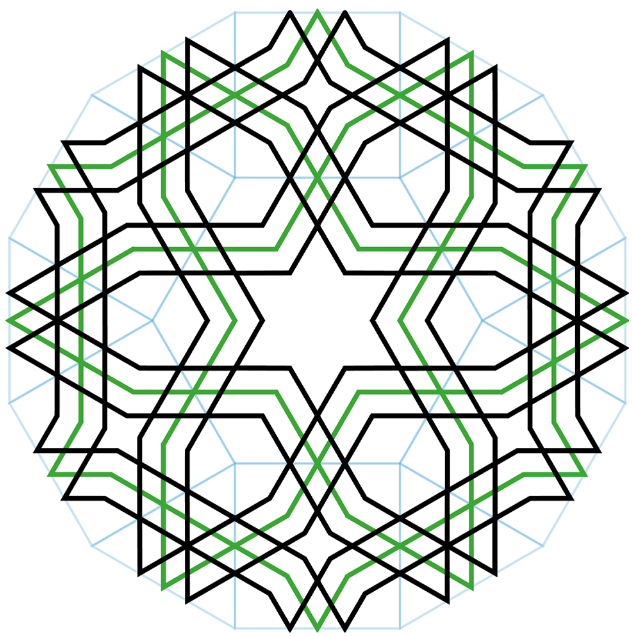
The complete pattern with the hexagonal tiling unit marked in red.



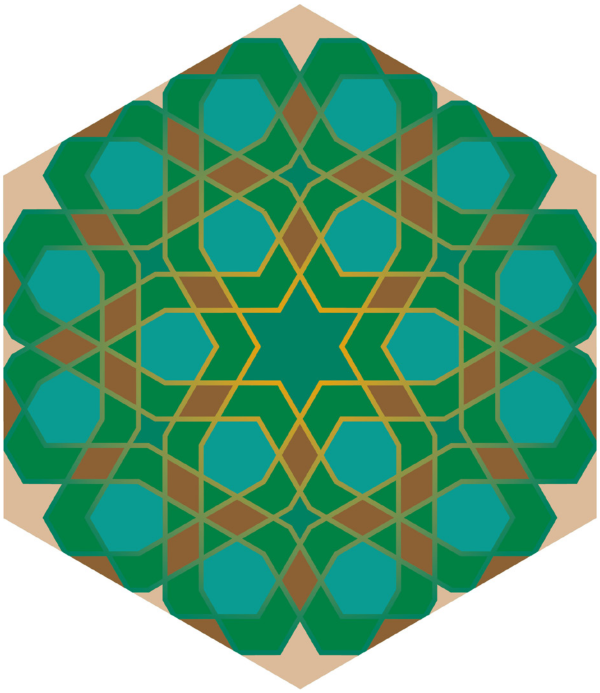
Here we can see the first pattern in green and the second in black. This makes the relationship between the two clear.

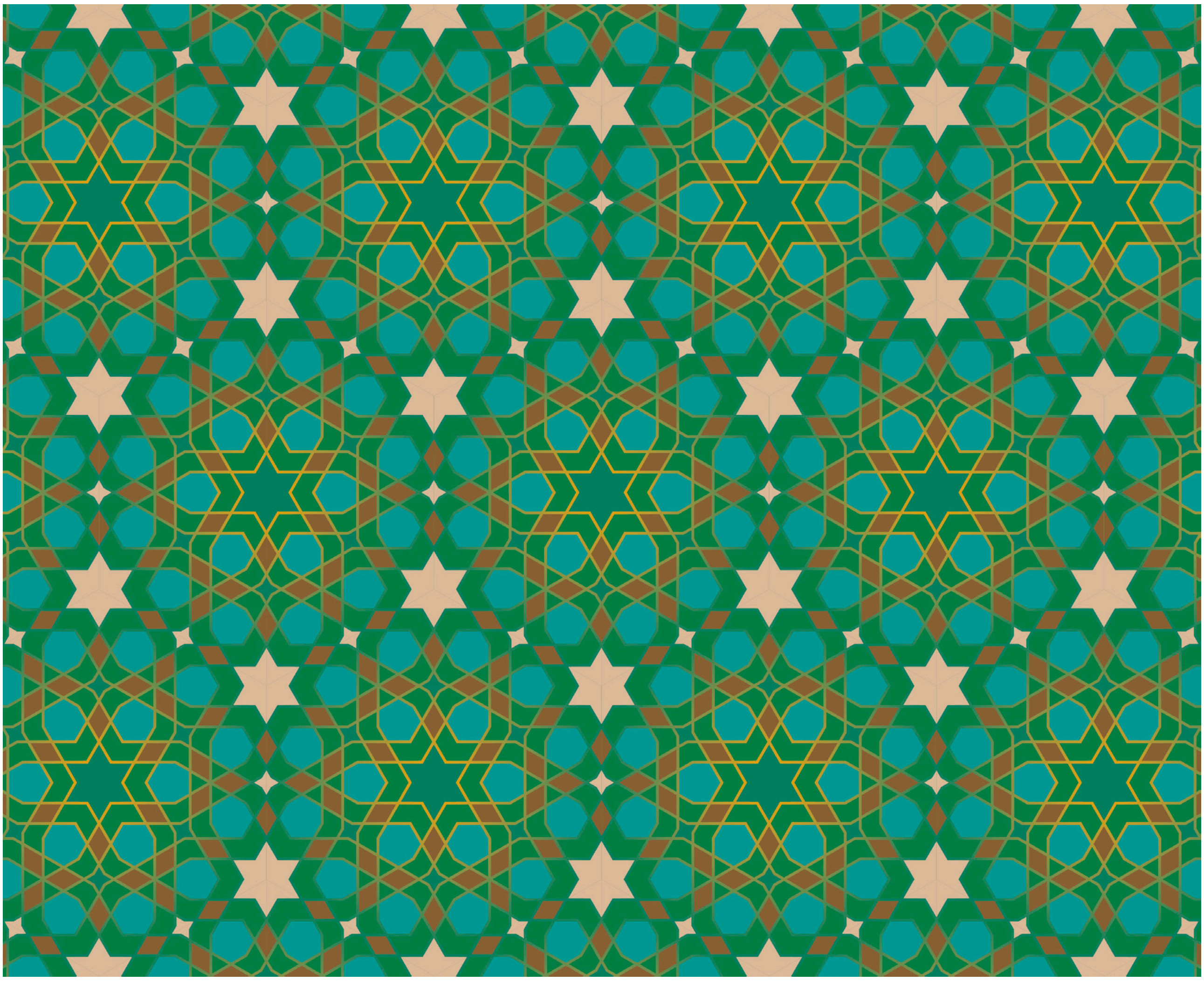
The green pattern involves lines crossing the edges of each polygon at the midpoint. The black pattern involves lines crossing along polygon edges twice at equidistant points (one- and two-thirds of the way along).

In both cases, the lines meet at an angle of 60°, a value which arises naturally from a six-fold division of the circle.



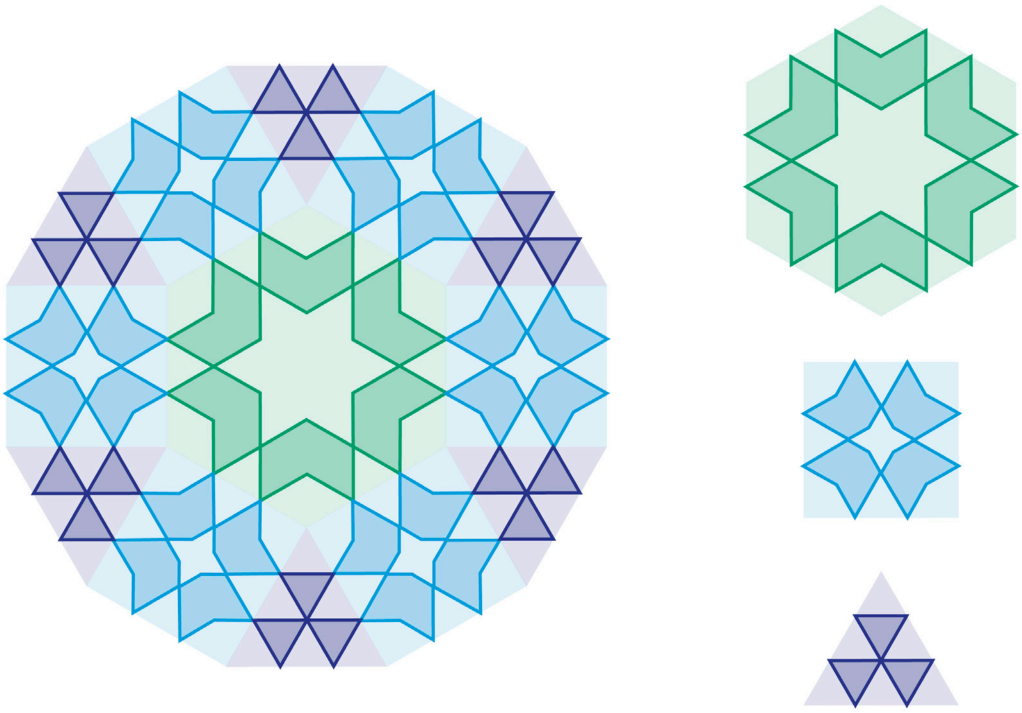
Here is an extended hexagonal tile which has been tessellated to create the repeating pattern below.



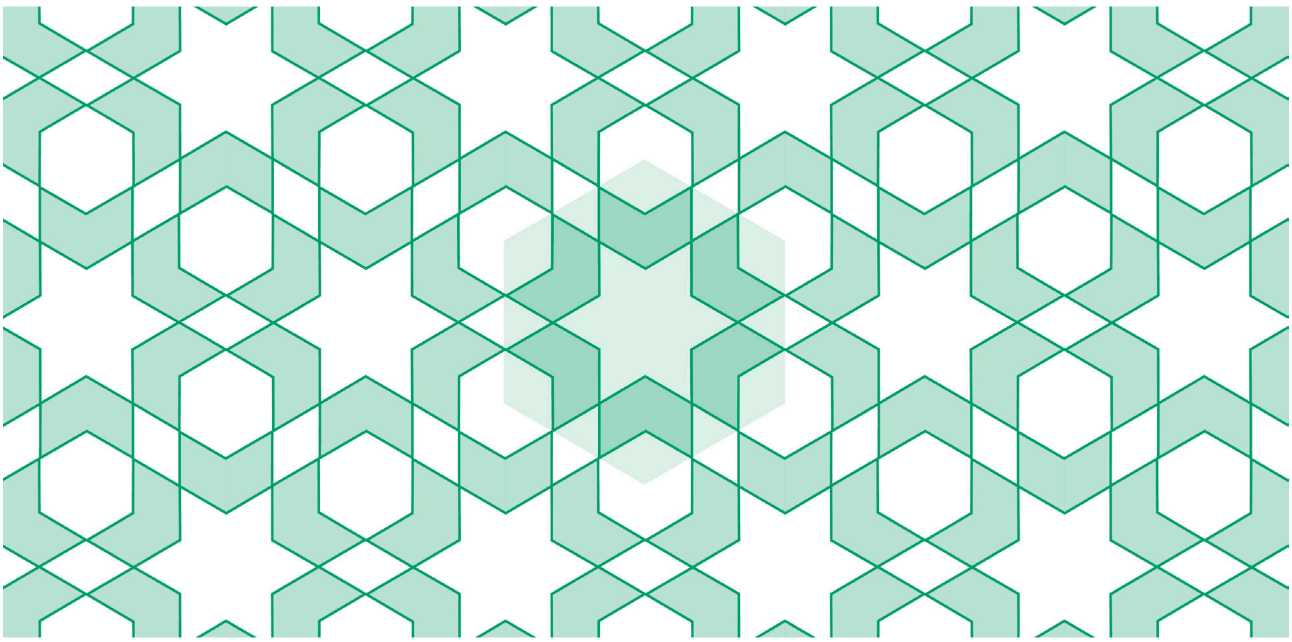


5. Exploring further tiling possibilities

The three tiles shown below can be extracted from the pattern we have drawn. These tiles can be configured in numerous ways which give rise to a wealth of pattern possibilities. The tiles have equal edge lengths and will join together seamlessly as they contain lines meeting at the same angle, the same distance along each edge.

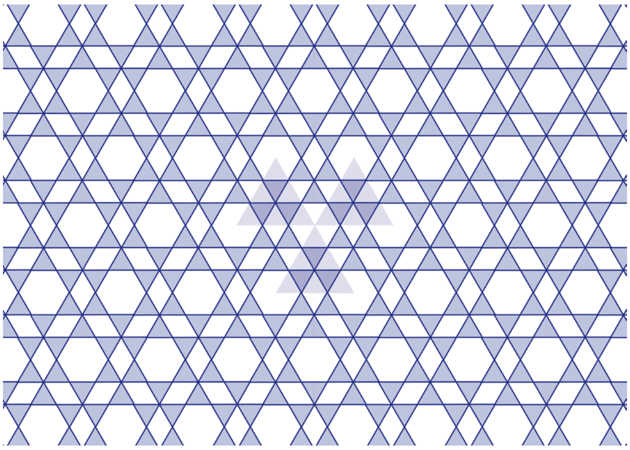


The three tiles

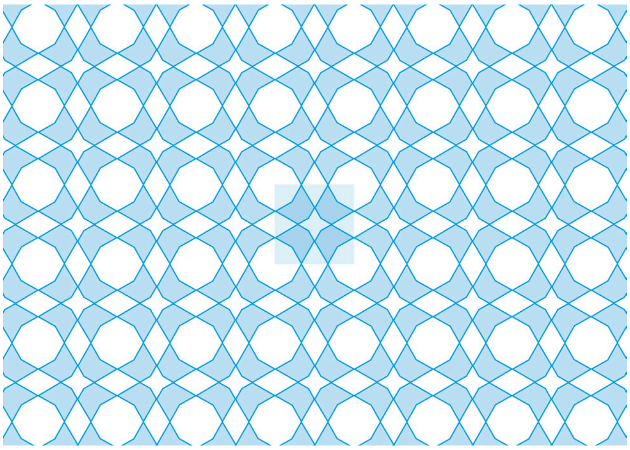


By tessellating just the hexagonal tile, we get the pattern seen in the Blue Mosque.

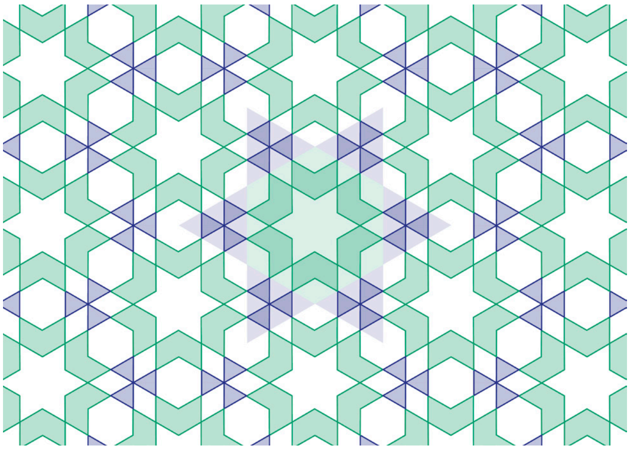
By applying a consistent construction method to a polygonal subgrid, we have generated a set of compatible tiling units which allow for the exploration of further configurations of the tiles as shown below.



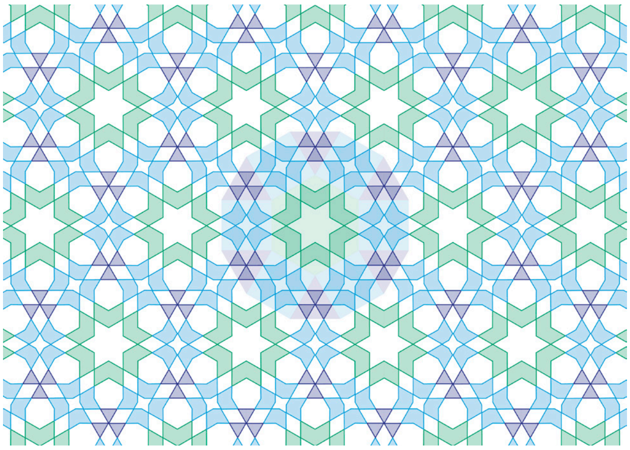
Created by tessellating just the triangular tile.



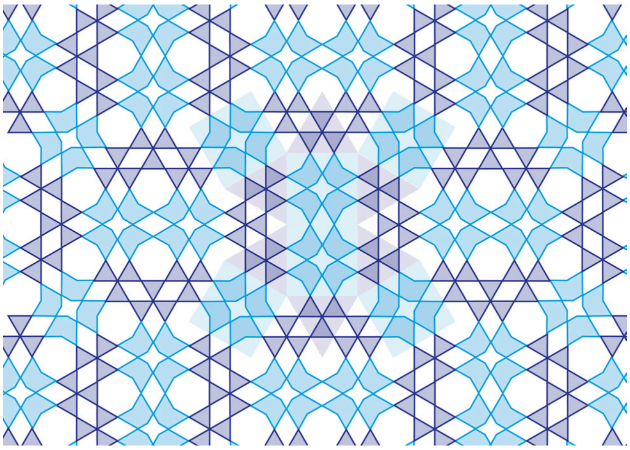
Created by tessellating just the square tile.



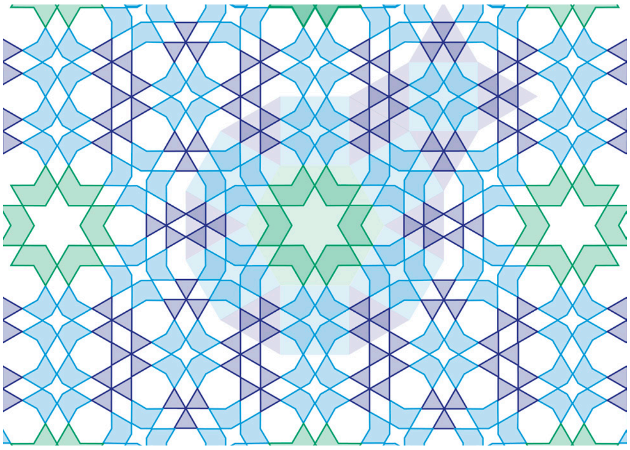
A combination of the hexagonal and triangular tiles.



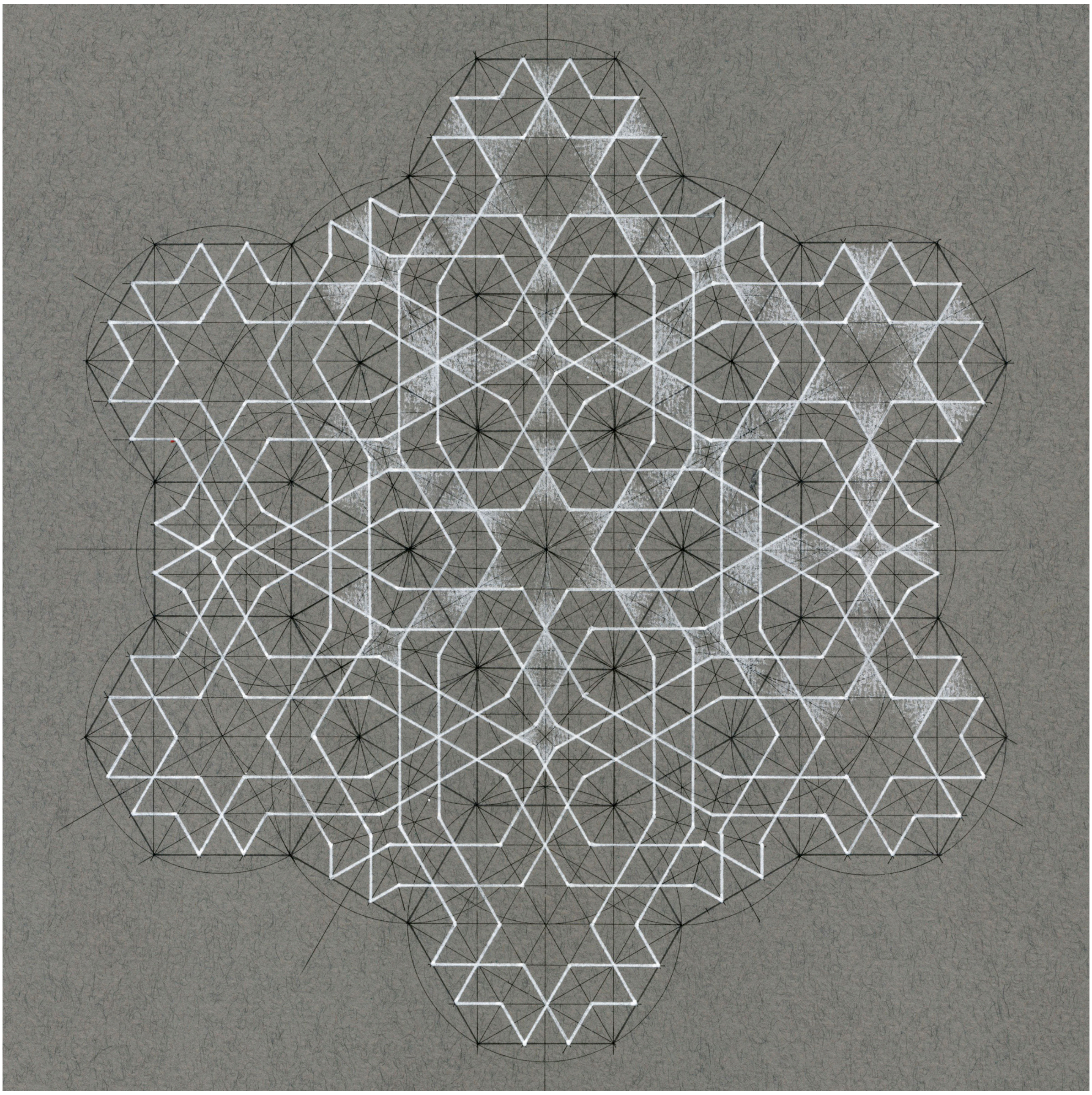
An extended tiling of the Archimedean subgrid.



A combination of the square and triangular tiles.



Another configuration of all three tiles.



Citations

[1] Majewski M., Sketch 19: Sultan Ahmed Mosque mosaic revisited, Posted 18.07.15

https://symmetrica.wordpress.com/2015/07/18/sketch-19-sultan-ahmed-mosque-mosaic-revisited/

[2] Bodner B.L., Hankin's 'Polygons in Contact' Grid Method for Recreating a Decagonal Star Polygon Design, Bridges 2008.